

Problem Set 1: Due TBA

1) Complete the calculation of the induced mass of potential flow around a sphere, which was begun and discussed in class. In particular, show the energy of potential flow is

$$E = \rho \left[4\pi(\mathbf{A} \cdot \mathbf{u}) - V_0 \frac{u^2}{2} \right] = m_{ik} \frac{u_i u_k}{2},$$

where \mathbf{A} is the dipole moment of the flow and V_0 is the volume of the body in motion at \mathbf{u} .

Compute m_{ik} , the induced mass tensor. What is its value for a sphere?

2) Consider a small body immersed in a fluid flow which oscillates. Derive the general relation between the velocity of the body and that of the fluid. What is the result for a spherical body of density ρ_0 ?

3) Derive the energy relation

$$\frac{\partial}{\partial t} \left(\rho \frac{v^2}{2} + \rho \epsilon \right) = -\nabla \cdot \left(\rho \mathbf{v} \left(\frac{v^2}{2} + \omega \right) \right)$$

from the continuity, Euler and energy equations. Here, ω is the enthalpy density.

4) a) Derive the dispersion relation for an azimuthally symmetric wave propagating along the \hat{z} axis and in radius in an ideal incompressible, unbounded fluid rotating at $\mathbf{\Omega} = \Omega_0 \hat{z}$.

b) Now assume the fluid is bounded by a cylindrical wall at $r = R$. What is the profile of radial velocity?

5) A spherical hole with radius a is suddenly formed in an incompressible fluid filling all space. How long does it take for fluid to fill up the hole?